THERMAL SLIP OF A MODERATELY DENSE GAS ALONG A FLAT SURFACE

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The solution is constructed for the problem of thermal slip of a moderately dense gas along a flat surface. The method of half space moments is used.

Thermal slip has been studied in many papers (see [2], for example). As a rule, the Boltzmann equation with a model collision integral in the BGK form [4] has hence been used. The influence of the gas not being ideal on the thermal slip velocity is taken into account here by using the Chapman – Enskog equation for compact gases converted within the scope of the BGK ideas.

Let us consider a gas which is above a wall in temperature gradient field tangential to the wall. Let us introduce a Cartesian coordinate system with origin on the wall surface, x axis along the normal to the wall, and y axis along the wall surface in the direction of grad T.

The well-known Chapman – Enskog equation for dense gases [1] with a nonlocal collision integral, which is ordinarily expanded in a power series in the small parameter σ/L (σ is the effective molecular diameter, and L is the characteristic dimension of the problem), with only terms not above the first order in σ/L retained:

$$(\mathbf{v}\cdot\nabla) f = \chi \iint (f'f_1' - ff_1)\, \sigma^2 \mathbf{g}\cdot\mathbf{k} d\mathbf{k} d\mathbf{v}_1 + \chi \iint \mathbf{k}\,(f'\nabla f_1' + f\nabla f_1)\, \sigma^3 \mathbf{g}\cdot\mathbf{k} d\mathbf{k} d\mathbf{v}_1 + \frac{1}{2} \iint \mathbf{k}\cdot\nabla\chi\,(f'f_1' + ff_1)\, \sigma^3 \mathbf{g}\cdot\mathbf{k} d\mathbf{k} d\mathbf{v}_1 \tag{1}$$

is the initial equation. Here $\mathbf{g} = \mathbf{v}_1 - \mathbf{v}$ is the relative velocity of the gas molecules; k, a vector along the line of centers; χ, a factor taking account of the increase in the collision probability with the rise in gas density. The following expression for χ can be used for gases of moderate density:

$$\chi = 1 + \frac{5}{8}\, b\rho,$$

where $b = 2/3 \cdot \pi\sigma^3/m$; $\rho = mn$; n is the number of molecules per unit volume and m is the mass of the molecules.

In this case the characteristic dimension is the Knudsen layer thickness which equals the molecule mean free path λ in order of magnitude. The ratio σ/λ is therefore a small parameter. The requirement of smallness of σ/λ imposes a constraint on the density. Thus, if it is assumed that $\sigma/\lambda \sim 0.1$, then we obtain $n \approx 8.9 \cdot 10^{21}$, which corresponds to a pressure on the order of 300 atm (for hydrogen).

Let us introduce the following notation:

$$f^{eq} = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \exp[-(\mathbf{c} - \mathbf{G})^2], \quad \mathbf{c} = \left(\frac{m}{2kT}\right)^{\frac{1}{2}} \mathbf{v}, \quad \mathbf{G} = \left(\frac{m}{2kT}\right)^{\frac{1}{2}} \mathbf{u},$$

$$n = \int_{-\infty}^{\infty} f d\mathbf{v}, \quad \mathbf{u} = \frac{1}{n}\int_{-\infty}^{\infty} \mathbf{v} f d\mathbf{v}, \quad \frac{3}{2} kT = \frac{1}{n}\int_{-\infty}^{\infty} \frac{mv^2}{2} f d\mathbf{v}.$$

Since the influence of the wall on the molecule velocity distribution has a finite radius of action, the distribution function far from the wall should go over into the Chapman – Enskog distribution

$$f = f^{eq}[1 + \psi(\mathbf{c},\, y)],$$

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where

$$\psi(c, y) = \frac{1}{n}\left(\frac{2kT}{m}\right)^{\frac{1}{2}} A c_y S^{(1)}_{3/2} \frac{\partial}{\partial y} \ln T;$$

$$A = \frac{1}{\chi}\left(1 + \frac{3}{5} b\rho\chi\right) \frac{3\eta_0}{2kT};$$

$S^{(1)}_{3/2} = (5/2) - c^2$; η_0 is the gas viscosity at the same temperature under normal pressure. The viscosity of a dense gas is associated with η_0 by the relation

$$\eta = \eta_0 b\rho\left(\frac{1}{b\rho\chi} + \frac{4}{5} + 0.76 b\rho\chi\right).$$

Near the wall it is necessary to distinguish between the distribution functions of the incident and reflected molecules, which we denote by the superscripts − and +, respectively.

Let us seek the distribution function in the form

$$f^{\pm} = f^{eq}[1 + \psi(c, y) + \varphi^{\pm}(c, x)]. \tag{2}$$

Here φ is the correction to the distribution function, which takes care of the influence of the wall. As is shown in [3], $|\partial\varphi/\partial y| \ll |\partial\varphi/\partial x|$, hence φ can be considered a function of just c and x.

The main assumption of the BGK method is that the distribution function goes over into a local Maxwell distribution f^{eq} during one collision, hence, the substitution $f', f'_1 \to f'^{eq}, f'^{eq}_1$ must be made in all the integrals in the right-hand side of (1). Moreover, the first of the integrals is replaced by the expression $\nu(f^{eq} - f)$, where ν is the collision frequency. Let us also note that n, and therefore χ, vary slightly within the Knudsen layer limits. Taking the above into account, we substitute (2) into (1) while retaining first-order terms in σ/λ here:

$$(\mathbf{v}\cdot\nabla) f^{eq} + f^{eq}(\mathbf{v}\cdot\nabla)\varphi = -\nu f^{eq}(\psi + \varphi) + \int\int \mathbf{k}\cdot\nabla\chi f^{eq} f^{eq}_1 \sigma^3 \mathbf{g}\cdot\mathbf{k} d\mathbf{k} d\mathbf{v}_1 +$$
$$+ \chi\int\int f^{eq} f^{eq}_1 \mathbf{k}\cdot\nabla\varphi_1 \sigma^3 \mathbf{g}\cdot\mathbf{k} d\mathbf{k}\cdot d\mathbf{v}_1 + \chi\int\int f^{eq} f^{eq}_1 \mathbf{k}\cdot\nabla \ln f'^{eq}_1 f'^{eq}_1 \sigma^3 \mathbf{g}\cdot\mathbf{k} d\mathbf{k}\cdot d\mathbf{v}_1. \tag{3}$$

It is here taken into account that $f'^{eq} f'^{eq}_1 = f^{eq} f^{eq}_1$.

The first and third integrals in the right-hand side of (3) are easily evaluated analytically [1]; hence taking into account that the continuity equation and the momentum and energy conservation laws are satisfied far from the wall, we obtain

$$\frac{m}{kT}\left(1 + \frac{2}{5} b\rho\chi\right) v_x v_y \frac{\partial u}{\partial x} + v_y\left(1 + \frac{3}{5} b\rho\chi\right)\left(c^2 - \frac{5}{2}\right)\frac{\partial}{\partial y}\ln T$$
$$+ v_x \frac{\partial\varphi}{\partial x} = -\nu(\psi + \varphi) + \chi\int\int f^{eq}_1 \mathbf{k}\cdot\nabla\varphi_1 \sigma^3 \mathbf{g}\cdot\mathbf{k} d\mathbf{k}\cdot d\mathbf{v}_1. \tag{4}$$

We obtan an expression for ν

$$\nu = \frac{2nkT}{3\eta_0}\chi$$

from the condition that ψ is the Chapman – Enskog correction far from the wall.

Terms corresponding to the Chapman – Enskog solution

$$2\left(1 + \frac{2}{5} b\rho\chi\right) c_x c_y \frac{\partial G}{\partial x} + c_x \frac{\partial\varphi}{\partial x} = -\nu^*\varphi + \chi\left(\frac{2kT}{m}\right)^{-\frac{3}{2}} \int\int f^{eq}_1 \mathbf{k}\cdot\nabla\varphi_1 \sigma^3 \mathbf{g}^*\cdot\mathbf{k} d\mathbf{k} d\mathbf{v}_1, \tag{5}$$

$$\nu^* = \left(\frac{m}{2kT}\right)^{\frac{1}{2}}\nu, \quad \mathbf{g}^* = \left(\frac{m}{2kT}\right)^{\frac{1}{2}}\mathbf{g}$$

vanish in (4) for such a selection of ν. Let us introduce the new function $\Phi = 2c_y G + \varphi$, where we seek $\Phi^{\pm}$ in the form of a series expansion in Sonine polynomials in velocity space:

$$\Phi^{\pm} = a_0^{\pm} c_y + a_1^{\pm} c_y S^{(1)}_{3/2},$$

$$\Phi = \frac{a_0^+ + a_0^-}{2} c_y + \frac{a_0^+ - a_0^-}{2} c_y \operatorname{sign} c_x + \frac{a_1^+ + a_1^-}{2} c_y S^{(1)}_{3/2} + \frac{a_1^+ - a_1^-}{2} c_y S^{(1)}_{3/2} \operatorname{sign} c_x,$$

$$\operatorname{sign} c_x = \begin{cases} 1, & c_x \geqslant 0 \\ -1, & c_x < 0 \end{cases}, \qquad a_0^{\pm} = a_0^{\pm}(x), \quad a_1^{\pm} = a_1^{\pm}(x),$$

$$f^{eq} = f^0 (1 + 2c_y G), \quad f^0 = n \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} \exp(-c^2).$$

We rewrite (2) in the form

$$f^{\pm} = f^0 [1 + \psi_{,}(\mathbf{C}\ y) + \Phi^{\pm}(\mathbf{c},\ x)] \tag{6}$$

which it is easy to use to obtain an expression for G:

$$G = \frac{1}{4} (a_0^{+} + a_0^{-}).$$

Upon substitution of the expressions for Φ and G into (5), the integral in the right-hand side is easily evaluated and vanishes identically. We finally obtain

$$c_x \frac{\partial}{\partial x} (\Phi + 2\beta c_y G) = \nu^* (2c_y G - \Phi). \tag{7}$$

Here $\beta = (2/5) b\rho\chi \approx (2/5) b\rho$ since $b\rho$ is a small quantity for a moderate density gas so that terms of order higher than the first can be neglected. We linearize with respect to $b\rho$ where necessary, in all the formulas obtained below, unless specially stipulated otherwise.

To determine Φ uniquely, we introduce a boundary condition on the wall surface

$$f^{+}(c_x,\ c_y,\ c_z,\ 0) = q f^0 + (1 - q) f^{-}(-c_x,\ c_y,\ c_z,\ 0), \tag{8}$$

where q is the accommodation coefficient ($0 \leq q \leq 1$). Let us multiply (7) successively by $c_y (1 \pm \operatorname{sign} c_x) \exp(-c^2)$, $c_y S_{3/2}^{(1)} (1 \pm \operatorname{sign} c_x) \exp(-c^2)$ and let us integrate over velocity space. We obtain a system of moment equations

$$\frac{db_0^{\pm}}{dx} = -\frac{13}{24} \nu^* \sqrt{\pi} (b_0^{+} - b_0^{-}) \mp \frac{5}{12} \nu^* \sqrt{\pi}\, a_1^{\pm}, \tag{9}$$

$$\frac{da_1^{+}}{dx} = -\frac{1}{12} \nu^* \sqrt{\pi} (b_0^{+} - b_0^{-}) \mp \frac{5}{6} \nu^* \sqrt{\pi}\, a_1^{\pm} \cdot$$

Here $b_0^{\pm} = a_0^{\pm} + \beta/4 (a_0^{+} + a_0^{-})$.

We seek the solution of system (9) in the form

$$b_0^{\pm} = c_1 + \alpha_0^{\pm} c_2 \exp(-\alpha x), \quad a_1^{\pm} = \alpha_1^{\pm} c_2 \exp(-\alpha x).$$

Here

$$\alpha = \left(\frac{55}{72} \pi \right)^{-\frac{1}{2}} \nu^*, \quad \alpha_0^{+} = 1, \quad \alpha_0^{-} = 0.3736, \quad \alpha_1^{+} = 1.2834, \quad \alpha_1^{-} = 0.0306\,.$$

It is now necessary to go from the variables $b_0^{\pm}$ to the variables $a_0^{\pm}$. To do this it is sufficient to solve the system

$$(1 + \beta/4) a_0^{\pm} + \beta/4\, a_0^{\mp} = c_1 + \alpha_0^{\pm} c_2 \exp(-\alpha x).$$

We obtain

$$a_0^{\pm} = (1 - \beta/2) c_1 + [\alpha_0^{\pm} - (1 + \alpha_0^{-}) \beta/4] c_2 \exp(-\alpha x).$$

The constants c_1 and c_2 are determined from the boundary condition (8)

$$c_1 = \frac{A}{(1 - \beta/2)} \frac{1 - (1 - q) \alpha_0^{-}}{\alpha_1^{+} - (1 - q) \alpha_1^{-}} \left[1 - \frac{q (1 + \alpha_0^{-})}{1 - (1 - q) \alpha_0^{-}} \beta/4 \right] \frac{d}{dy} \ln T,$$

$$c_2 = A \frac{q}{(1 - q) \alpha_1^{-} - \alpha_1^{+}} \frac{d}{dy} \ln T.$$

Recalling that $G = (1/4)(a_0^{+} + a_0^{-})$, an expression for the slip velocity is easily obtained: $u_{sl} = (2kT/m)^{\frac{1}{2}} \lim\limits_{x \to \infty} G = (2kT/m)^{\frac{1}{2}} \frac{c_1}{2} (1 - \beta/2)$. Substituting the expression for c_1 here we finally obtain

$$u_{sl}=\frac{3}{2}\mu\frac{1-(1-q)\alpha_0^-}{\alpha_1^+-(1-q)\alpha_1^-}\left[1-\frac{2+q+(3q-2)\cdot\alpha_0^-}{1-(1-q)\alpha_0^-}b\rho/10\right]\frac{d}{dy}\ln T,\qquad \mu=\eta/\rho. \tag{10}$$

In the limit cases of pure diffusion (q = 1) and pure specular (q = 0) reflection, we obtain

$$u_{sl}=1.69\mu(1-0.337b\rho)\frac{d}{dy}\ln T, \tag{11}$$

$$u_{sl}=\frac{3}{4}\mu(1-0.2b\rho)\frac{d}{dy}\ln T. \tag{12}$$

Formulas (10)-(11) differ from the corresponding formulas for a rarefied gas by terms proportional to $b\rho$. As $\rho \to 0$, expressions (10)-(12) go over into the corresponding expressions in [2].

Let us determine the thermal slip coefficient as follows:

$$u_{sl}=k_{sl}\mu\frac{d}{dy}\ln T.$$

Then it follows from (10)-(12) that the thermal slip coefficient k_{sl} is less in dense than in rarefied gases.

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PECULIARITIES IN THE ONE-DIMENSIONAL MODEL OF RADIANT HEAT EXCHANGE

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Radiant heat exchange is considered in a one-dimensional model. The role of internal heat transfer is considered. Maximum and minimum heat liberation values are determined. A method for calculation is proposed.

The most widely used model for study of radiant heat exchange in a furnace is the one-dimensional model. In such a model the furnace operating space is likened to a channel, along which the exhaust gases move. The gas temperature along the directions perpendicular to the motion is assumed constant. There is no theoretical justification for the use of such a model.

We will write the energy equation of an elementary volume in the following form:

$$-\frac{W}{f}\frac{dT}{dz}+q_c=\frac{h_{V-F}^{r}(z)}{f}\varepsilon_s\sigma_0(T^4-T_s^4)-\frac{1}{f}\int_0^l q_{it}(z,z_k)\,dz_k, \tag{1}$$

where

$$h_{V-F}^{r}(z)=\frac{H_{V-F}^{r}(z,H)}{\Delta z},\quad \Delta z\to 0; \tag{2}$$

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